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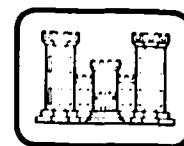UNCLASSIFIED CRREL-80-18   NL

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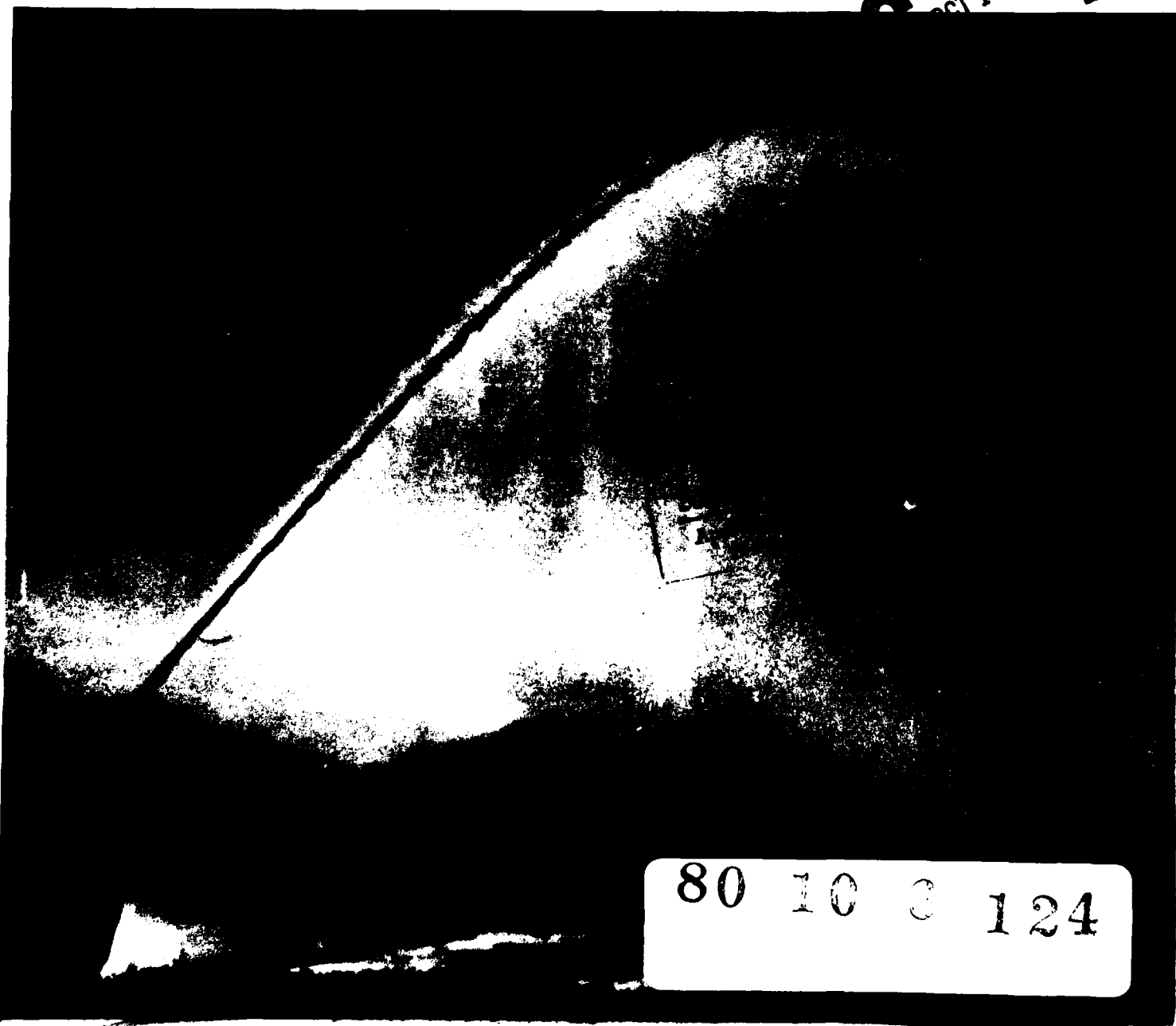
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Heat and mass transfer from freely falling drops at low temperatures

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Cover: Spraying water to accelerate ice growth at
Fort Wainwright, Alaska, during winter of
1977-78. (Photograph by P. Johnson.)

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Heat and mass transfer from freely falling drops at low temperatures

John P. Zarling

August 1980

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The use of ice as a structural material is common practice for certain applications in cold regions. Techniques such as surface flooding or water spraying are used to accelerate ice growth rates, thereby lengthening the winter construction season. This report examines the heat and mass transfer rates from freely falling water drops in cold air. Design equations which predict the amount of supercooling of the drops as a function of outdoor ambient temperature, drop size and distance of fall are given.

PREFACE

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NOMENCLATURE

a, b	Coefficients in humidity/temperature relationship
A	Surface area of drop
B, C	Constants defined in text
C_p	Specific heat
C_D	Drag coefficient
d	Drop diameter
D	Mass diffusivity
F	Force
h	Enthalpy
h_c	Convective heat transfer coefficient
h_d	Convective mass transfer coefficient
h_f	Enthalpy of liquid water
h_g	Enthalpy of water vapor
h_r	Radiation heat transfer coefficient
h_{fg}	Enthalpy of vaporization
k	Thermal conductivity
m	Mass flow rate
T	Temperature
V	Volume of drop
U	Velocity
W	Humidity ratio

Dimensionless numbers

$\text{Le} = h_c/(\rho_a C_{p,a} h_d)$	Lewis number
$\text{Nu} = (h_c d)/K$	Nusselt number
$\text{Pr} = \nu/\alpha$	Prandtl number
$\text{Re} = Ud/\nu$	Reynolds number
$\text{Sc} = \nu/D$	Schmidt number
$\text{Sh} = (h_d d)/D$	Sherwood number
α	Thermal diffusivity
β	Constant defined in text
γ	Constant defined in text
ϵ	Emissivity
σ	Stefan-Boltzmann constant
ρ	Density
μ	Absolute viscosity
ν	Kinematic viscosity
τ	Time

Subscripts

a	Air
c	Thermal convection
d	Mass convection
D	Drop
e	Evaporation
f	Film condition or saturated liquid
fg	Vaporization
g	Saturated vapor
i	Conditions entering
r	Radiation
s	Saturation
v	Drops in control volume
w	Water
o	Conditions leaving

HEAT AND MASS TRANSFER FROM FREELY FALLING DROPS AT LOW TEMPERATURES

John P. Zarling

INTRODUCTION

Ice has been used numerous times as a structural material in cold regions to substantially reduce construction costs for roads, bridges, and work pads. To shorten construction time, techniques such as surface flooding or spraying of water have been used in the past, as reported by Hoffman (1967) and McFadden (1976), to accelerate ice growth. However, very little engineering design information exists related to ice growth rates that can be achieved using these artificial thickening methods. The rapid ice building project currently being conducted by the Alaskan Projects Office of CRREL is an attempt to provide an engineering design methodology which will predict the rates of artificial ice thickening.

The approach being taken by the Alaskan Projects Office is two-fold: experimental and analytical. A pump–nozzle system (see cover photo) has been designed, assembled, and used to pump 0°C water from beneath the ice sheet and jet it into the air. As the water jet breaks up into drops, a large surface area/volume ratio results which produces high heat transfer rates and subsequent supercooling of the drops as they fall back to the ice surface. In fact, field tests have shown that some drops actually nucleate into ice before striking the ice sheet. This report, however, will only examine the heat and mass transfer rates from freely falling drops on an analytical engineering design basis.

There have been several research programs, both theoretical and experimental, conducted on heat and mass transfer from drops. The bulk of these studies have cooling tower, spray cooling, or chemical processing applications as their focus. Ranz and Marshall (1952) experimentally investigated the evaporation rates from drops, their studies confirming the analogy between heat and mass transfer. They also presented correlative expressions for the Nusselt and Sherwood numbers in terms of Reynolds and Prandtl or Schmidt numbers. Yao and Schrock (1976) conducted a series of experiments on heat and mass transfer from freely falling drops over the zero to terminal velocity range. They modified the Ranz and Marshall expressions to include acceleration effects. Chao (1969) performed an analytical study of transient heat and mass transfer from translating drops at large Reynolds and Péclet numbers. He included internal circulation within the drop and performed a boundary layer similarity analysis at the drop surface to arrive at transient and steady-state Nusselt numbers.

The behavior of drops and sprays from a dynamic point of view has also received attention. Hughes and Gilliland (1952) presented a general review including the gross motion of drops and the detailed motion in and around individual drops. The emphasis of the article is on correlations in connection with the effect of acceleration on drag, the equilibrium distortion or oscillations from prolate to oblate spheroidal shapes, and the internal circulation caused by skin friction. Shattering of drops due to aerodynamic forces under both transient and steady state conditions was studied experimentally by Lane (1951). The largest drops he tested were 4 mm in diameter which required a steady relative air speed of 12 m/s before shattering occurred. Smaller drops required higher shattering velocities. (The free fall speed of 4-mm-diam water drops is less than the shattering velocity.) Lapple and Sheperd (1940) have presented equations and drag coefficients for calculating the trajectories of individual spherical particles undergoing accelerated motion due to gravity in a viscous fluid. Experimental tests also conducted by them compared the actual and theoretical trajectories, which were in fairly good agreement.
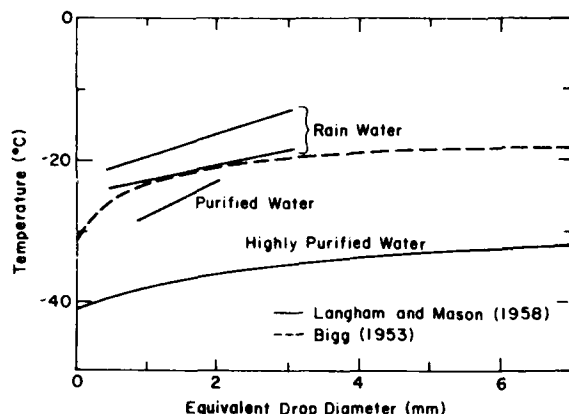
Figure 1. Nucleation temperature of water drops as a function of water purity and drop diameter.

The rate of evaporation from sprays has been reported by Dickinson and Marshall (1968). The principal parameters considered were the drop mean diameter and size distribution, initial drop velocity and temperature, and air velocity and temperature. Computations were carried out and results reported on the interrelationship of these parameters. Chen and Trezek (1977) introduced a wet bulb temperature weighting factor which was applied to the single drop thermal performance model to account for local temperature and humidity variations within the spray field.

Nucleation and dendritic ice formation within water drops falling through low temperature air are related to the formation of ice fog produced by water vapor output from automobile exhaust, power plant stacks, residential chimneys, power plant cooling ponds and other sources associated with an urban environment. Vapor fog from these sources nucleates to an ice fog whenever the ambient temperature approaches $-30°C$. Benson (1970) presented an overall review of the ice fog phenomenon including its sources, distribution, and physical nature in interior Alaska. A section of his report discusses the supercooling and nucleation of small water drops into ice. An extensive review of nucleation temperatures of supercooled water drops has been conducted by Langham and Mason (1958). Figure 1 shows that the nucleation temperature of water drops decreases with decreasing drop size and increasing water purity until the homogeneous nucleation temperature, $-40°C$, is reached. Once nucleation has occurred, the drop temperature rises to $0°C$ as the latent heat of fusion liberated during ice formation warms the supercooled water. This results in an increased heat transfer rate after nucleation due to the increased temperature difference.

Another application of this analysis is the purification of salt water (sea water) by spraying water into the air under winter arctic conditions (Peyton and Johnson 1967). The ice dendrites formed following nucleation of the falling drops are fresher or less saline than the salt water. After the ice-water drops land, the brine drains away, leaving a mass of low salinity ice particles that might be used as a domestic water source.

The remainder of this report has been sub-divided into three sections: the velocity problem, heat and mass transfer from a single drop, and heat and mass from a system of drops. The solution to the velocity problem is required for the calculation of the heat and mass transfer rates for both the single drop and system of drops. The single drop solution provides insight into the maximum rates of heat transfer and mass transfer. However, this model does not account for the changes in the air temperature and humidity as the drops fall. These effects have been included in the model of a system of drops which results in a decrease in the cooling rate of the drops as the ambient air temperature and humidity surrounding the drops increase. In practice, spraying water with a sweeping motion of the nozzle will aid in enhancing the cooling rate from the drops as compared to a fixed nozzle arrangement, especially during calm wind conditions.

VELOCITY PROBLEM

In order to predict the heat and mass transfer rates from the water drops, the velocity of the drops must be determined. The water jet trajectory obeys the laws of Newtonian dynamics. However, the problem is complicated by the fact that the jet breaks up into individual drops near the apex of its path due to the interactions of inertial, aerodynamic and surface tension forces. In the horizontal direction, the speed of the drops will decrease continuously from the point of discharge at the nozzle to the point where the drops strike the ice due to aerodynamic drag. In the vertical direction, the jet will have its maximum speed at the nozzle, decrease to zero velocity at the apex of the trajectory where the jet breaks up, and then the drops will increase in speed to terminal velocity as they fall downward.

The equation of motion for an individual drop of diameter d in vector notation (Lapple and Shep-erd 1940) can be written as

$$\frac{d\vec{U}}{d\tau} + \frac{3}{4}\frac{C_D}{d}\left(\frac{\rho_a}{\rho_w}\right)|\vec{U}|\vec{U} + \vec{g}\left(\frac{\rho_a}{\rho_w}-1\right) = 0 \tag{1}$$

where U = velocity

τ = time

C_D = drag coefficient that is a function of its Reynolds number

d = drop diameter

g = gravitational acceleration

ρ_a = air density

ρ_w = water drop density.

At terminal velocity in the y direction, $dU_y/d\tau$ is zero and the resulting equation can be solved for U_y as

$$U_y = \sqrt{\frac{4dg(\rho_w-\rho_a)}{3\rho_a C_D}} . \tag{2}$$

For liquid droplets, the following drag coefficient correlations are available (Chen and Trezek 1977),

$$C_D = \frac{24}{Re} + \frac{6}{1+\sqrt{Re}} + 0.27 \quad 1 < Re \leqslant 1000 \tag{3}$$

and

$$C_D = 0.6649 + 0.2712\times10^{-3}Re + 1.22\times10^{-7}Re^2 - 10.919\times10^{-12}Re^3 \tag{4}$$

for $1000 < Re < 3600$, where Re is the Reynolds number.

Based on eq 2 and 3 or 4, terminal velocities of drops of varying diameter falling vertically have been calculated at an air temperature of $-18°C$ and are listed in Table 1.

In all subsequent calculations in this report, the velocity of the freely falling drops is assumed to be equal to their terminal speed in the vertical or y direction. Reynolds numbers will also be based on the terminal velocity. Field observations of jetting water into the air supports this assumption, since the horizontal velocity component of the drops appears very small beyond the apex of the jet, especially at large vertical angles.

Table 1. Terminal speed vs drop diameter.

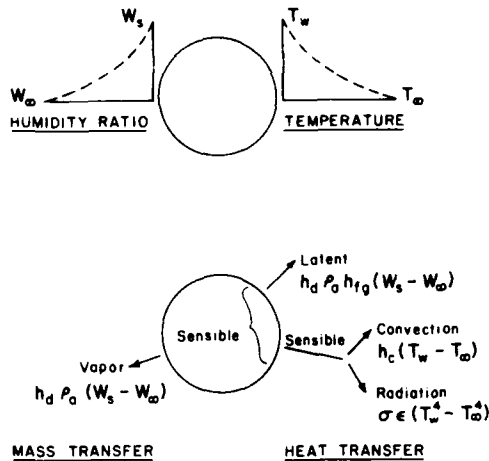| D Drop diameter (mm)	C_D Drag coefficient	U Terminal velocity (m/s)	Re Reynolds number
0.5	1.129	2.07	87
1.27	0.583	4.54	477
2.54	0.495	6.95	1463
3.81	0.577	7.86	2489
5.08	0.727	8.11	3417
6.35	0.889	8.20	4317

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Figure 2. Heat and mass transfer be-
tween water drop and ambient air.

HEAT AND MASS TRANSFER PROBLEM—A SINGLE DROP

There are two approaches which have been used in the past to analyze the heat and mass transfer rates from a freely falling drop. The first approach is to assume that the internal motion within the drop is so vigorous that complete mixing occurs. The temperature profile is flat and the thermal and mass transfer resistances occur only at the drop surface. The alternate approach is to assume no mixing in the drop so that the energy equation reduces to a transient heat flow problem. Intuitively, the mixing model will yield answers indicating a higher heat flux than the nonmixing model.

The governing differential equation for the heat and mass transfer from a falling, well-mixed drop, as shown in Figure 2, is

$$\rho_w C_p V \frac{dT_w}{d\tau} = -A[h_c(T_w - T_\infty) + h_d \rho_a h_{fg}(W_s - W_\infty) + \sigma\epsilon(T_w^4 - T_\infty^4)] \tag{5}$$

where C_p = specific heat of water
$\quad V$ = drop volume
$\quad A$ = surface area of drop
$\quad h_c$ = a convective heat transfer coefficient
$\quad T_w$ = water drop temperature
$\quad T_\infty$ = ambient temperature
$\quad h_d$ = a convective mass transfer coefficient
$\quad \rho_a$ = air density
$\quad \rho_w$ = water density
$\quad h_{fg}$ = enthalpy of vaporization of water
$\quad W_s$ = saturation humidity ratio
$\quad W_\infty$ = ambient humidity ratio
$\quad \sigma$ = Stefan–Boltzmann constant
$\quad \epsilon$ = emissivity.

The term on the left-hand side of the above equation represents the time rate of change of energy of the drop as it falls. The three terms on the right-hand side of this equation represent thermal energy transports by convection, evaporation and radiation, respectively.

In order to solve this differential equation, the heat and mass transfer coefficients will be assumed constant. Furthermore, the fluid physical properties will also be assumed invariant. The radiation

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term in the governing equation can be simplified by introducing the radiation heat transfer coefficient h_r which is defined as

$$h_r = \sigma\epsilon(T_w^2 + T_\infty^2)(T_w + T_\infty).$$ (6)

For small temperature differences, h_r can be approximated as a constant or

$$h_r = \sigma\epsilon(T_0^2 + T_\infty^2)(T_0 + T_\infty)$$ (7)

where T_0 is the initial drop temperature. An analysis of McFadden's (1976) data on arctic cooling pond water surface temperatures, air temperatures, and net outgoing radiation indicates that the ambient air temperature closely approximates the effective sky temperature. Therefore, the ambient air temperature was chosen as the effective sky temperature to which the drop radiates.

Next the evaporation term must be expressed in terms of temperature. Since the saturation humidity ratio is a function of temperature, a parabolic curve using temperature as the independent variable can be used to fit humidity ratio data at saturation over the appropriate temperature range,

$$W_s = aT_w^2 + bT_w + c,$$ (8)

or in terms of the humidity ratio potential,

$$W_s - W_\infty = a(T_w^2 - T_\infty^2) + b(T_w - T_\infty).$$ (9)

The coefficients in the above equations have been determined over three temperature ranges using a least-squares fit of the humidity ratio at saturation conditions and are given below:

$$W_s = 6.0948 \times 10^{-6} T_w^2 + 2.7723 \times 10^{-4} T_w + 3.7653 \times 10^{-3} \qquad -20°C \leq T \leq 0°C \qquad \pm 4\%$$

$$W_s = 5.1896 \times 10^{-6} T_w^2 + 2.6184 \times 10^{-4} T_w + 3.7382 \times 10^{-3} \qquad -25°C \leq T \leq 0°C \qquad \pm 8\%$$

$$W_s = 4.4299 \times 10^{-6} T_w^2 + 2.4589 \times 10^{-4} T_w + 3.7002 \times 10^{-3} \qquad -30°C \leq T \leq 0°C \qquad \pm 14\%.$$

If a more accurate representation of the humidity ratio-temperature function is desired, a higher order polynomial can be used or the quadradic equation can be "patched" over smaller temperature intervals.

There are two approaches which can be used to arrive at the mass transfer coefficient. The first uses the Lewis number, Le, to relate the forced convective heat and mass transfer coefficients for water in air (Threlkeld 1970), or

$$Le = \frac{h_c}{\rho_a c_{p,a} h_d} \cong \left(\frac{\alpha}{D}\right)^{2/3}$$ (10)

where α is the thermal diffusivity and D the mass diffusivity. For a water-air system, α/D is approximately 0.85, from which the value of the convective mass transfer coefficient h_d can be easily calculated from known correlations for the convective heat transfer coefficient. The second and seemingly more direct approach is to use the Ranz and Marshall (1952) convective heat and mass transfer coefficients for drops which have been correlated from experimental data as

$$Nu = \frac{h_c d}{k} = 2 + 0.6 Pr^{1/3} Re^{1/2}$$ (11)

and

$$Sh = \frac{h_d d}{D} = 2 + 0.6 Sc^{1/3} Re^{1/2}$$ (12)

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where Nu = Nusselt number
Pr = Prandtl number
Sh = Sherwood number
Sc = Schmidt number.

Equation 5 can now be rewritten as

$$\frac{dT_w}{d\tau} = \frac{-6}{\rho_w C_p d} \left| (h_c + h_r + h_d \rho_a h_{fg} b)(T_w - T_\infty) + h_d \rho_a h_{fg} a(T_w^2 - T_\infty^2) \right| . \tag{13}$$

To simplify the algebra in the above equation, let

$$B = \frac{6(h_c + h_r + h_d \rho_a h_{fg} b)}{\rho_w C_p d} ; \qquad C = \frac{6\rho_a h_d h_{fg} a}{\rho_w C_p d} ; \qquad 0 = T_w - T_\infty .$$

After introducing these definitions into eq 13 and rearranging, the following equation results:

$$\frac{d0}{d\tau} = -B0 - C0(0 + 2T_\infty). \tag{14}$$

Integrating the above equation and then applying the initial condition that $0 = 0_i$, $(T_w = T_i)$, at $\tau = 0$ yields the solution

$$0 = \frac{\gamma 0_i}{\gamma e^{\gamma \tau} + C0_i(1 - e^{\gamma \tau})} \tag{15}$$

where $\gamma = B + 2T_\infty C$.

The amount of supercooling of drops initially at $0°C$ ranging in size from 0.5 mm to 6.5 mm and falling through ambient air at $-18°C$ was calculated based on eq 15 with the results shown in Figure 3. As expected, the smaller drops cool to a lower temperature when falling a given distance as compared to larger drops. This fact is chiefly due to the larger volume/surface area ratio of larger drops and the increased falling time of small drops. The ratio of energy lost by the drop to the latent heat of fusion is easily estimated as $C_p(T_f - T_D)/L$ where T_f, T_D and L are the freezing temperature, final drop temperature, and latent heat of fusion, respectively.

The relative amounts of cooling by convection, radiation and evaporation from the drop can be determined by evaluation of these respective terms in eq 5. For a 5-mm-diam drop falling at terminal speed at $0°C$ in a saturated $-18°C$ ambient air environment, convective heat transfer is 68%, radiative heat transfer is 2% and evaporative heat transfer is 30%.

In order to compare the effectiveness of jetting water into the air to surface flooding, the heat transfer rates of the two techniques must be evaluated. The heat loss for the surface flooding case
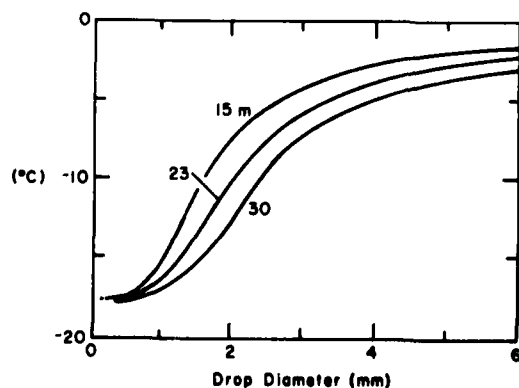

Figure 3. Cooling curves for water crops falling from various heights in a -18°C environment.

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can be determined by following McFadden's (1976) recommendations on heat transfer equations for calculating heat loss from cooling ponds in low temperature environments. The Rimsha-Donchenko equation for predicting the free convective heat loss from open water during winter is given as

$$Q_c = |3.9 + 0.17(T_w - T_a)| (T_w - T_a) \qquad (W/m^2). \qquad (16)$$

Dalton's equation in modified form to account for evaporative heat loss at low ambient temperatures is presented as

$$Q_e = 21,820(W_s - W_a) \qquad (W/m^2). \qquad (17)$$

The radiative loss can be predicted in an identical manner as was done for the drops, or

$$Q_r = \sigma\epsilon(T_w^4 - T_a^4) \qquad (W/m^2). \qquad (18)$$

Assuming no wind, a water temperature of $0°C$, and an ambient air temperature of $18°C$, the water surface heat losses by convection, evaporation, and radiation are 125 W/m^2, 65 W/m^2, and 89 W/m^2, respectively. For a 4-mm-diam drop at $0°C$ falling at terminal speed, convective, evaporative, and radiative heat loss rates are 1024 W/m^2, 375 W/m^2, and 89 W/m^2, respectively. Added to this is the fact that the drops provide a much larger exposed surface area per unit volume of water than does ponded water, which greatly enhances the cooling of the sprayed water and subsequently increases the ice formation rate. For example, a 5-cm layer of water on the ice surface would contain the equivalent of approximately 1.5 million 4-mm-diam drops per meter^2 of surface area. This yields a drop surface to water surface area ratio of 75 to 1.

HEAT AND MASS TRANSFER—A SYSTEM OF DROPS

Figure 4 shows a schematic diagram of the column of cold air and a falling water spray. Assuming steady-flow conditions, an energy balance on a differential volume element yields

$$m_a dh = - [m_w - m_a(W + dW - W_i)] \, dh_f + m_a h_f dW \qquad (19)$$

which can be simplified to
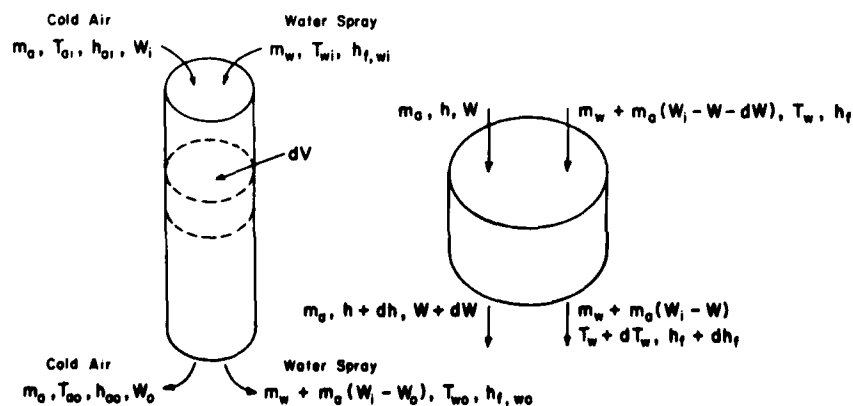
$$m_a dh = -m_w dh_f + m_a h_f dW \qquad (20)$$

Figure 4. Schematic diagram of energy-mass balance across the air column and differential element.

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by neglecting the second order terms in eq 19. In eq 20, m_a is the mass flow rate of air, m_w is the mass flow rate of water, dh is the change in enthalpy of the air, dh_f the change in enthalpy of the water, and dW the change in humidity ratio of the air across the differential volume. The change in energy of the water in the control volume is due to convective heat and mass transfer from the drops or

$$-m_w dh_f = h_c A_v dV (T_w - T_a) + \rho_a h_d A_v dV (W_s - W) h_{fg}. \tag{21}$$

The term on the left-hand side of the above equation represents the change in energy of the water, and the two terms on the right-hand side of the equation represent the heat transfer due to convection and evaporation, respectively. In this equation, h_c and h_d are the heat and mass transfer coefficients, A_v is the surface area of water drops contained in the control volume, T_w and T_a are the water drop and air dry bulb temperatures, and h_{fg} is the enthalpy of evaporation of the water spray. The change in concentration of water vapor in air must be equal to the mass transfer from the drops or

$$m_a dW = \rho_a h_d A_v dV (W_s - W) \tag{22}$$

where W_s and W are the humidity ratios of the saturated and moist air streams. Upon substituting the Lewis number, $Le = h_c / h_d \rho_a C_{p,a}$ into eq 21, the following equation is obtained

$$-m_w dh_f = \rho_a h_d A_v dV \left[Le C_{p,a} (T_w - T_a) + (W_s - W) h_{fg} \right]. \tag{23}$$

Substituting eq 20 and 22 into eq 23 and rearranging yields

$$\frac{dh}{dW} = Le\, C_{p,a} \frac{(T_w - T_a)}{(W_s - W)} + h_g. \tag{24}$$

Now using the approximation of constant specific heat $C_{p,a}$ in the defining equation for the enthalpy of the air stream, or

$$h = C_{p,a} T_a + 2501 W \tag{25}$$

eq 25 can then be used to write

$$C_{p,a}(T_w - T_a) = (h_s - h) - 2501(W_s - W). \tag{26}$$

Equation 26 is now substituted into eq 24 to yield

$$\frac{dh}{dW} = Le\, \frac{(h_s - h)}{(W_s - W)} + (h_g - 2501\, Le). \tag{27}$$

This equation represents the operating or condition line on the psychrometric chart for the changes in state as water drops pass through the column of air. Although data for Le for this application are not available, Threlkeld (1970) recommends an approximate relationship for air flow given previously by eq 10. Thermal and vapor diffusivity data for dry and saturated moist air can be found in Eckert (1959). An accurate although approximate solution to eq 27 can be obtained graphically on a psychrometric chart. For one set of operating conditions for the air column, assume that the entering water temperatures T_{wi}, the water flow rate m_w, the air flow rate m_a, and the initial air temperature T_{ao} and pressure p_a are known. Then eq 27 can be solved for dh/dW, the slope of the condition or operating line on the psychrometric chart. A line segment having this slope is then drawn on the chart through the point representing the entering state of the air. A short distance along this line a new state "1" is arbitrarily located. Then, rewriting eq 20 as

$$\Delta T_w = -\frac{m_a}{m_w C_p} (\Delta h - h_f \Delta W) \tag{28}$$

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the water temperature $T_{w,1}$ corresponding to state "1" can be calculated. Next eq 27 is resolved for $(dh/dW')_1$ and the procedure repeated until the final air temperature is determined. Performing an energy balance for the entire air column provides a check on the final air temperature determined from the condition line on the psychrometric chart, or

$$m_a h_{ai} + m_w h_{1,i} = m_a h_{ao} + |m_w - m_a(W_o - W_i')| h_{1,o}. \tag{29}$$

Rearranging yields

$$h_{ao} = h_{ai} + \frac{m_w C_p}{m_a}(T_{wi} - T_{wo}) + (W_o' - W_i')h_{1,o}. \tag{30}$$

If the leaving air temperature has been determined and the average mass transfer coefficient $h_d A_v$ is known, then the volume of air may be obtained from

$$V = \frac{m_a}{h_d A_v} \int_{W_i'}^{W_o'} \frac{dW'}{W_s - W'}. \tag{31}$$

The above integral can be solved numerically or graphically. (One convenient graphical method is the use of the Stevens diagram.)

For this study, a computer program was written (App. A) to calculate leaving air temperature and humidity ratio, leaving water temperature, and percent moisture added based on the foregoing analysis and the calculation procedure presented below. The input variables to the computer program are the entering air temperature, humidity ratio and velocity, the entering water temperature, drop size and velocity, flow ratio and air column height. The program begins by assuming a leaving water temperature at the bottom of the column. Then based on the given entering air conditions and this assumed leaving water temperature, the saturation humidity, thermal diffusivity, viscous diffusivity, mass diffusivity, and thermal conductivity are calculated at these conditions (-50°C to +20°C) according to the following formulas:

$$W_s = 0.003894 e^{0.0944427 T_w} \qquad \text{(kgw/kga)} \tag{32}$$

$$\alpha = 1/(57736 - 585.78 T_a) \qquad \text{(m}^2\text{/s)} \tag{33}$$

$$\nu = 1/(80711.7 - 766.15 T_a) \qquad \text{(m}^2\text{/s)} \tag{34}$$

$$D = 2.227 \times 10^{-5} \left(\frac{T_a + 273}{273}\right)^{1.81} \qquad \text{(m}^2\text{/s)} \tag{35}$$

$$k = 0.024577 + 9.027 \times 10^{-5} \ T_a \qquad \text{(W/m C°)} \tag{36}$$

These equations were developed using a linear least-squares fit of property data given in Karlekar (1977) for α, ν, and k, and W_s values were obtained from ASHRAE (1977). The mass diffusivity relationship is given in Eckert (1959).

In order to calculate the heat and mass transfer coefficients for the water drops required for eq 22 and 27, the Prandtl, Pr, Schmidt, Sc, and Reynolds, Re, numbers are calculated based on the previously defined property data. The Nusselt number, Nu, for heat transfer and Sherwood number, Sh, for mass transfer cannot be calculated according to Ranz (1952), eq 11 and 12. At this point ΔW across the differential element can be calculated according to eq 22. Then using this value, Δh for the air stream can be determined using eq 27. Finally, ΔT_w can be found using eq 28. The humidity ratio, air stream enthalpy, and water temperature are then incremented to determine the conditions on the other side of the differential element. These conditions become the initial conditions for the second element as the process is continued step by step downward until the bottom of
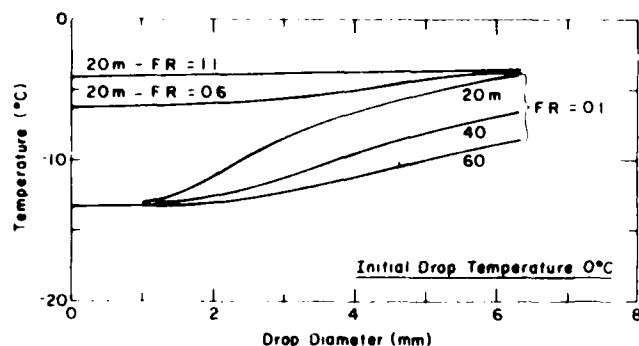
Figure 5. Cooling curves for a system of water drops falling from various heights and at various flow ratios (F.R.) in a -18°C environment.
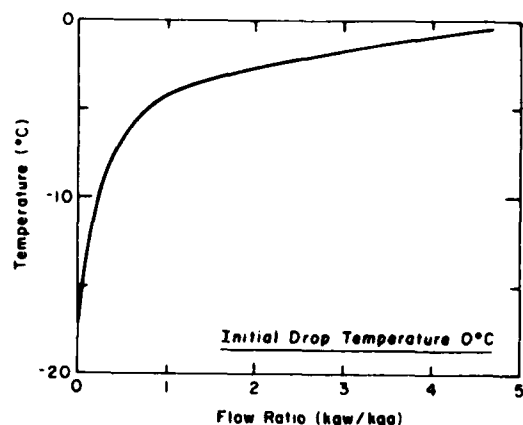

Figure 6. Final temperature of water drops falling 20 m in a -18°C environment as a function of flow ratio.

the column is reached. The accuracy of the conditions calculated by this procedure at the bottom of the column can then be checked by using eq 30. If greater accuracy is desired, smaller step sizes (volume increments) can be chosen and the entire procedure repeated.

Cooling curves for a system of water drops falling from various heights in a -18°C environment are shown in Figures 5 and 6. In Figure 5 the effects of falling distance and drop size on the final temperature of the drops are shown for a flow ratio of 0.1 (kgw/kga). As anticipated, drops falling a greater distance cool more. Also, as the size of the drops decreases, the thermal equilibrium condition between drops and the air is approached such that drops falling 20 m or 60 m have the same final temperature. This is due to the fact that equilibrium is reached before the drops travel 20 m so that the air temperature and drop temperature are equal and no further cooling occurs. Also, since the drops are smaller, their time of fall is greater, providing more time for cooling to take place. Cooling curves for flow ratios of 0.6 and 1.1 are also shown in this figure. As expected, when the flow ratio increases, the cooling of the drops decreases since more water is contained within the air column. This fact is clearly shown in Figure 6 in which final drop temperature as a function of flow ratio is presented.

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APPENDIX A. FORTRAN IV PROGRAM TO CALCULATE FINAL DROP TEMPERATURE, AIR
TEMPERATURE, AND HUMIDITY.

Program is written in British engineering units.

```
10    REAL LE
20    DATA IXT/"Y"/
30    104 CONTINUE
40    PRINT,"     HEAT AND MASS TRANSFER FROM DROPS"
50    PRINT, " "
60    PRINT, "AIR COLUMN HEIGHT (FT)"
70    READ, SH
80    PRINT, "INLET AIR TEMPERATURE (F)"
90    READ, TGI
100   PRINT, "INLET AIR VELOCITY (FPS)"
110   READ, VG
120   PRINT, "INLET HUMIDITY RATIO (LBW/LBA)"
130   READ, WI
140   PRINT, "DROP DIAMETER (IN)"
150   READ, D
160   D = D/12.
170   PRINT, "SPRAY VELOCITY (FPS)"
180   READ, VW
190   PRINT, "INLET WATER TEMPERATURE (F)"
200   READ, TWI
210   PRINT, "INITIAL FLOW RATIO (LBW/LBA)"
220   READ, FI
230   PRINT, "FINAL FLOW RATIO (LBW/LBA)"
240   READ, FF
250   FF = FF+.01
260   PRINT, ""
270   FR = FI
280   70 CONTINUE
290   20 X = SH/50
300   TG1 = TGI
310   WG1 = WI
320   TW1 = TWI
330   ND = FR*6/(3.1416*D**3)/62.4
340   AV = ND*3.1416*D*D*39.715/(TGI+460)
350   ID = 0
360   NEND = 50
370   DO 80 J=1, NEND
380   CW = 1.0
390   TA = (TG1+TW1)/2.
400   DI = D*12.
410   VW = .057350+413.2885*DI-2719.527*DI*DI+9164.955*DI**3-12738.54*
420&  DI**4
430   VR = VW+VG*(TG1+460)/(TGI+460)
440   VK = .4542439/3600*EXP(.0039815*TA)
450   TD = (.6357178+.0025373*TA)/3600
460   PR = VK/TD
470   RE = VR*D/VK
480   TK = (.013115 + 2.63929E-5*TA)/3600
490   HC = TK*(2.0+.6*PR**.333*SORT (RE))/D
500   DEN = 39.715/(TG1+460)
510   IF(ID.EO.1) GO TO 102
520   101 DC = (2.397E-4)*((TA+460)/460)**1.81
530   GO TO 103
540   102 DC = (3.86779E-9)*((TA+460)*5./9.)**1.9415
```

13

```
550    103 CONTINUE
560    SC = VK/DC
570    CP = .24+.45*WG1
580    HD = DC*(2.0+.6*SC**.333*SORT(RE))/D
590    LE = HC/(HD*DEN*CP)
600    WS = .0007265*EXP(.0524685*TW1)
610    HWG = 1075+.45*TG1
620    HG1 = (.24+.45*WG1)*TG1+1075*WG1
630    HGSW = (.240+.45*WS)*TW1+1075*WS
640    DWG = DEN*HD*AV*X*(WS-WG1)
650    HFW2 = CW*(TW2-32)
660    DHG = DWG*(LE*(HG1-HGSW)/(WG1-WS)+HWG-1075*LE)
670    DTW = +DWG/(FR*CW)*HFW2-DHG/(FR*CW)
680    TW2 = TW1 + DTW
690    WG2 = WG1 + DWG
700    HG2 = HG1 + DHG
710    TG2 = (HG2-1075*WG2)/(.24+.45*WG2)
720    TG1 = TG2
730    WG1 = WG2
740    TW1 = TW2
750    80 CONTINUE
760    D = D*12.
770    PRINT, "DROP DIAMETER =", D
780    PRINT, "FLOW RATIO =", FR
790    PRINT, "FINAL WATER TEMPERATURE (F)", TW2
800    PRINT, "LEAVING AIR TEMPERATURE (F)", TG2
810    PRINT, "FINAL HUMIDITY RATIO (LBW/LBA)", WG2
820    PRD = ((WG2-WI)/WI)*100.
830    PRINT, "PERCENT WATER ADDED", PRD
840    PRINT," "
850    FR = FR+.5
860    D = D/12.
870    IF (FR.LT.FF) GO TO 70
880    PRINT,"NEW DATA ?  YES-Y OR NO-N"
890    READ(5,105) IANS
900    105 FORMAT(A1)
910    IF(IANS.EQ.IXT) GO TO 104
920    STOP
930    END
```

END

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